

systems, the Arabic abjad is multiplicative only in combination with 1000. Most likely, the Fez numerals are an unusual blend of the Greek, *zimām*, and Arabic alphabetic systems adopted among a very unusual group of users, highly educated Arabized Christians living in Muslim-dominated southern Spain.

The Fez numerals did not last long in Spain; I know of no texts from the fourteenth century or later in which they are used. They were described by the Moroccan mathematician Ibn al-Banna in the early fourteenth century, and later by the great historian Ibn Khaldun in the late fourteenth century, both of whom lived and worked at Fez (Colin 1933: 206; Guergour 1997: 68). Their use in Morocco began in earnest only in the sixteenth century, however, after the expulsion of the Moors from Spain in 1492. They were used frequently throughout the sixteenth and seventeenth centuries in accounting and other commercial contexts, after which time they began to be replaced by the Arabic positional numerals. While they were known to scholars, their use was always mercantile and legal, never mathematical or astronomical (Guergour 1997: 74). The Fez numerals were still used in the early twentieth century for indicating monetary values in wills and in related legal documents (Colin 1933, Sanchez Perez 1935). Because the meaning of the numerals was known to only a few learned notaries by this time, the system had become a cryptographic notation to prevent fraudulent modifications and forgeries, and generally to restrict access to information (Colin 1933: 195). Political changes in post-colonial Morocco have ended this system's use.

ARMENIAN

Before the introduction of Christianity, there was no native Armenian script, and the Babylonian, Greek, and Old Persian scripts were used for literary purposes. The Armenian adoption of Christianity in the early fourth century AD was followed by enormous influence from the Greek-speaking world. In the early fifth century AD (probably in 406 or 407), the Armenian scholar-monk Mesrop Mashtots (c. 360–440) developed the first uniquely Armenian script, an alphabet of thirty-six letters, in order to translate the Bible from Greek into Armenian (Sanjian 1996: 356).¹² At the same time, the letters of the alphabet were assigned numerical values as shown in Table 5.18.

These signs are the *erkat'agir* 'iron-forged letters' preferred from the fifth through thirteenth centuries AD, and still used for epigraphic inscriptions (Thomson 1989; Sanjian 1996: 357). In the tenth century, cursive letters known as *bolorgir* began to be used, and are the standard forms used in modern Armenian writing. The Armenian system is ciphered-additive and decimal, and is written from left to right.

¹² The modern Armenian script has thirty-eight letters, the last two of which (*o* and *fē*) were introduced in the medieval period and have no numerical value.

Table 5.18. Armenian *erkat'agir* numerals

	1	2	3	4	5	6	7	8	9
1s	Ա	Բ	Գ	Դ	Ե	Զ	Է	Ը	Թ
10s	Ճ	Ի	Լ	Խ	Ծ	Կ	Ն	Չ	Ղ
100s	Ճ	Մ	Յ	Ն	Շ	Ո	Չ	Պ	Ջ
1000s	Ռ	Ս	Վ	Տ	Ր	Ց	Ի	Փ	Ք
346 = ՅԽԶ									

Because the ancient Armenian alphabet had thirty-six letters, it had enough signs to express the complete series from 1000 to 9000 as well as all the units, tens, and hundreds. The system could thus denote any number less than 10,000. However, unlike many ciphered-additive alphabetic systems, the Armenian system does not use multiplication to express higher values, which were written in full using lexical numerals.

Very little epigraphic or paleographic evidence survives from the earliest centuries of the system's use. The Armenian numerals were probably developed on the model of the Greek alphabetic numerals, just as the Armenian script itself was derived from the Greek. Many other scripts have been suggested as possible ancestors of the Armenian script, based on resemblances in the shapes of certain characters (Gamkrelidze 1994: 37), while there are few resemblances to the Greek alphabet. However, of these likely ancestors, only the Greek alphabet used appropriate alphabetic numerals. Thus, regardless of the origins of the script-signs, the principle of alphabetic numeration was certainly borrowed from Greece. It is unclear whether the Armenian alphabetic numerals were developed by Mesrop Mashtots himself (or his assistants) in the early fifth century AD, or whether they were produced later in the century. Figure 5.2 is a monumental grave inscription from the temple of Garni east of Yerevan, which commemorates a ninth-century Armenian Catholicos, also named Mashtots; the first three signs are numerals (300 + 40 + 6), indicating his death-year to be 346 according to the Armenian calendar, or 897 AD.

Although a connection is sometimes asserted to exist between the Armenian and Georgian alphabetic numerals, the evidence for this is too tenuous to suggest any definite link. The primary similarities between the two are that they were used in the same region and had distinct signs for 1000 through 9000. The only system that is derived from the Armenian alphabetic numerals is the variant Armenian system developed in the seventh century AD by Anania Shirakatsi. The Armenian numerals did not spread beyond the limited area around Lake Van where Armenian was spoken, nor do they appear to have inspired the creation of any foreign

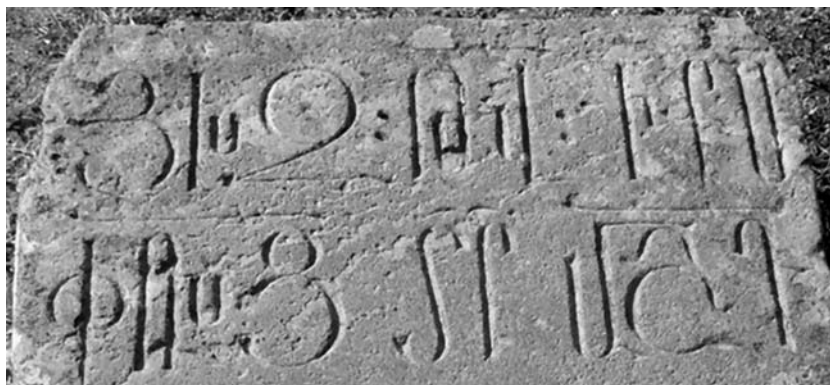


Figure 5.2. Grave inscription of the Armenian Catholicos Mashtots (897 AD); the first three visible signs of the inscription are 300 + 40 + 6, the year of his death. Courtesy Gabriel Kepekljian.

systems. After the development of the minuscule Armenian script, these signs were also used numerically in the same manner.

Ciphered-positional numerals – the Arabic system used by the neighboring Seljuk Turks – were first used in Armenia in the twelfth century (Shaw 1938–39: 368). Yet Armenian writers retained the alphabetic numerals for most ordinary purposes long afterward. Only in the mid seventeenth century, when Armenia had been firmly under Ottoman control for some time, did ciphered-positional numerals (Arabic, then later Western) replace the alphabetic system. Wingate (1930) has published an unusual, undated, and unsolved “magic square” arithmetical puzzle using both Arabic positional and Armenian alphabetic numerals, part of a scroll contained within a Armenian “family amulet” designed to be worn upon the person. The Armenian alphabetic system is still sometimes used for numbering chapters of the New Testament, although page and verse numbers are most often written using Western numerals. Otherwise, the numerals used in modern Armenia are the standard Western numerals.

SHIRAKATSU'S NOTATION

The Armenian astronomer, geographer, and mathematician Anania Shirakatsi¹³ was born ca. AD 595–600 and was most likely a monk in the Armenian Church (Hewsen 1968: 34). While little-known today outside his native country, Shirakatsi's contribution to Armenian learning is unparalleled, particularly his synthesis

¹³ Also known as Ananiah Shiragooni, or Ananias of Shirak.

Table 5.19. Armenian numerals: Shirakatsi's notation

	1	2	3	4	5	6	7	8	9
1s	Ա	Բ	Գ	Դ	Ե	Զ	Է	Ը	Թ
10	Ճ		100	Հ		1000	Ռ		

of Persian, Arabic, Greek, and other scientific knowledge. In addition to these accomplishments, Shirakatsi developed a very interesting numerical notation system in his “Book of Arithmetic” (*T'uabant'iwn*), which is a collection of arithmetical tables designed for the instruction of pupils. The basic form of this system uses twelve signs, as shown in Table 5.19 (Shaw 1938–39: 270).

The individual signs are identical to those used for the appropriate numbers in the traditional Armenian system. However, Shirakatsi showed how these signs could be combined to express numbers through multiplication as well as addition. In this system, a unit-sign followed by one of the three power-signs (for 10, 100, or 1000) indicates that the values of the two should be multiplied; these pairs of signs were combined into numeral-phrases through addition. Instead of writing 9642 as ԹՌԽԲ (9000 + 600 + 40 + 2), as in the traditional Armenian alphabetic numerals, Shirakatsi would write the same number as ԹՌ.ԶՀԴՃԲ (9 × 1000 + 6 × 100 + 4 × 10 + 2). Thus, where the traditional Armenian system is ciphered-additive, Shirakatsi's system is multiplicative-additive.

Any numeral-phrase can be written more compactly with the traditional alphabetic numerals than with Shirakatsi's variant – so why would Shirakatsi advocate its use? Firstly, it requires knowing fewer symbols (twelve versus thirty-six) in order to express any number less than 10,000. More importantly, numbers greater than 10,000 could be expressed using multiplicative combinations of two or three signs. To do so, however, one needs the entire repertoire of Armenian numerals from 1 through 9000, as described earlier. For numbers from 10,000 through 90,000, Shirakatsi juxtaposed the signs for 10 through 90 with the sign for 1000. Similarly, the numeral-phrases for 100,000 through 900,000 combined the signs for 100 through 900 with the sign for 1000. Alternatively, the hundred thousands could be expressed using unit-signs followed by a 100-sign and then a 1000-sign. Thus, one could write 460,000 as ԽԿՌ – (400 + 60) × 1000 – or ԴՀԿՌ – ((4 × 100) + 60) × 1000. This system is no longer a purely decimal system, but has a mixed base of 10 and 1000. For values below 1000, it is purely multiplicative-additive, but above 1000, the multiplicand that is juxtaposed with the sign for 1000 (Ռ) is not a single sign, but rather a ciphered-additive *numeral-phrase*. In the “Book of Arithmetic,” numbers up to the ten millions are expressed relatively compactly (Abgarian 1962: 46; Hewsen 1968: 42).

Shaw (1938–39: 369) believes that this system was not developed by Shirakatsi in the seventh century, but was a commonly used variant system, of which Shirakatsi's writings are the only surviving remnant. I do not believe there is any reason to regard the system as anything other than the creation of Shirakatsi himself, since its structure is never found in Greek, Syriac, Hebrew, or any other alphabetic system. Shirakatsi may have borrowed the notion of multiplicative structuring from one of two foreign sources. The numerals developed by the fifth century AD Indian mathematician Āryabhata (Chapter 6) were multiplicative-additive; it is possible that Shirakatsi, a mathematician with extensive knowledge of foreign writers, knew of Āryabhata's numerals and emulated them. Similarly, it is vaguely plausible that Shirakatsi knew of the classical Chinese multiplicative-additive numerical notation system (Chapter 8). Neither hypothesis has any direct evidentiary support.

Shirakatsi's system is thus a structurally innovative local variant of the Armenian numerals designed to facilitate the representation of large numbers of the sort that would be needed for his astronomical and mathematical calculations. There is no evidence that his system was adopted by any later writers, or that it had any effect on the development of other numerical notation systems throughout the world. Instead, we should view this system as the creative invention of a single individual, used only within his lifetime.

GEORGIAN

Like the Armenians, the Georgians developed a script and numerical notation system modeled after the Greek alphabet shortly after they converted to Christianity. While the creation of this first Georgian alphabet is often attributed in folklore to King Parnavaz in the third century BC, there is no direct evidence of Georgian writing until the fifth century AD, at which time the *asomtavruli* or majuscule script began to be used (Holisky 1996). More familiar to modern scholars, however, are the *mnedruli* characters developed in the eleventh century AD, which continue to be used to write the Georgian language today. The numerals associated with this script are shown in Table 5.20 (Holisky 1996: 366).

The system is decimal and ciphered-additive and, like the Georgian script, is written from left to right. Like the Armenian script, the Georgian script had enough letters to serve for all numerical values up to 9000. Some later inscriptions even include a special sign for 10,000 (Ⴀ). There is no evidence that the Georgian alphabetic numerals were ever used to express larger numbers than this, either through multiplication or through additional signs. Presumably, such numbers were always written out in full using lexical numerals.

There is an undeniable structural similarity between the Georgian and Armenian systems, which both, unlike the Greek alphabetic numerals, have enough